

CALCULUS BC

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the slope of the line tangent to the curve $x^2 + 2xy + 3y^2 = 2$ when $y = 1$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) -1

(D) 0

(E) $\frac{1}{8}$

2. $\int_{-1}^1 xe^{x^2} dx =$

(A) $-e$

(B) $-\frac{e}{2}$

(C) 0

(D) $\frac{e}{2}$

(E) e

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3. If, for $t > 0$, $x = t^2$ and $y = \cos(t^2)$, then $\frac{dy}{dx} =$

- (A) $\cos(t^2)$ (B) $-\sin(t^2)$ (C) $-\sin(2t)$ (D) $\sin(t^2)$ (E) $\cos(2t)$
-

4. The function $f(x) = 4x^3 - 8x^2 + 1$ on the interval $[-1, 1]$ has an absolute minimum at $x =$

- (A) -11 (B) -1 (C) 0 (D) 1 (E) $\frac{4}{3}$
-

5. $\int \frac{x \, dx}{x^2 + 5x + 6} =$

- (A) $\ln \left| \frac{(x+3)^3}{(x+2)^2} \right| + C$
(B) $\ln \left| (x+3)^3(x+2)^2 \right| + C$
(C) $\ln \left| \frac{(x+2)^2}{(x+3)^3} \right| + C$
(D) $\ln \left| (x+3)^2(x+2)^3 \right| + C$
(E) $\ln \left| (x+3)(x+2) \right| + C$
-

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6. $\frac{d}{dx}(x^2 \sin^2 x) =$

- (A) $2x \sin 2x$
 - (B) $2x \cos^2 x$
 - (C) $x \sin 2x$
 - (D) $2x \sin^2 x + x^2 \cos^2 x$
 - (E) $2x \sin^2 x + x^2 \sin 2x$
-


7. The line normal to the curve $y = \frac{x^2 - 1}{x^2 + 1}$ at $x = 2$ has slope

- (A) $-\frac{8}{25}$
 - (B) $-\frac{25}{8}$
 - (C) 1
 - (D) $\frac{8}{25}$
 - (E) -1
-

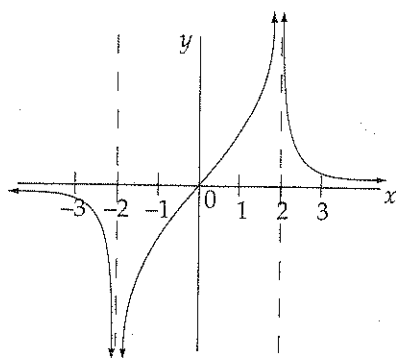
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8. If f and g are differentiable functions and $h(x) = f(x)e^{g(x)}$, then $h'(x) =$

- (A) $f'(x)e^{g'(x)}$
 - (B) $f'(x)e^{g(x)} + f(x)e^{g'(x)}$
 - (C) $e^{g(x)}[f'(x) + f(x)g'(x)]$
 - (D) $e^{g(x)}[f'(x) + 1]$
 - (E) $e^{g'(x)}[f'(x) + g'(x)]$
-

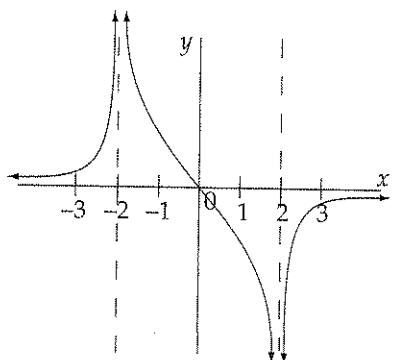
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9.

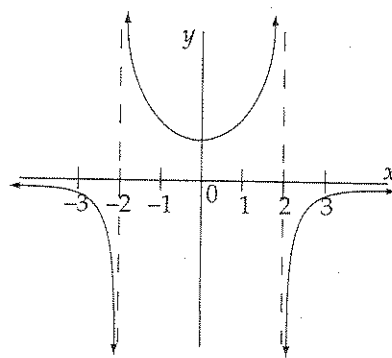


The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f'(x)$?

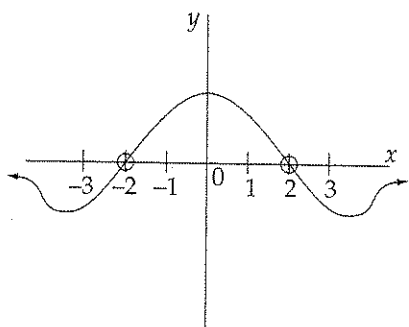
(A)



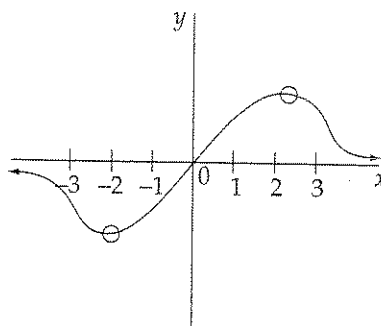
(D)



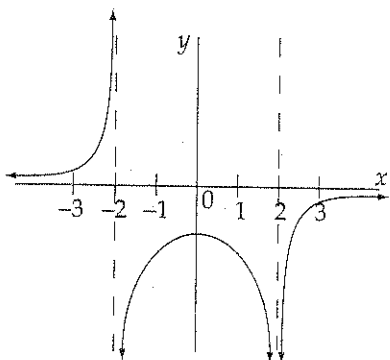
(B)



(E)



(C)



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10. $\int_e^{e^2} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx =$

- (A) $\frac{e^4}{2} + 2e^2 + 2$
(B) $\frac{e^2}{2} + 2e + 1$
(C) $e^4 + 2e^2 + e$
(D) $\frac{e^4}{2} + \frac{3e^2}{2} - 2e + 1$
(E) $\frac{e^4}{2} + e^2 + e$
-

11. $\int_4^{\infty} \frac{dx}{x^2 + 16}$

- (A) $\frac{\pi}{16}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) Divergent
-

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12. What is the equation of the line tangent to the graph of $y = \sin^2 x$ at $x = \frac{\pi}{4}$?

(A) $y - \frac{1}{2} = -\left(x - \frac{\pi}{4}\right)$

(B) $y - \frac{1}{2} = \left(x - \frac{\pi}{4}\right)$

(C) $y - \frac{1}{\sqrt{2}} = \left(x - \frac{\pi}{4}\right)$

(D) $y - \frac{1}{\sqrt{2}} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y - \frac{1}{2} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

13. If $f(x) = \begin{cases} ax^2 + 3ax + 5; & x \geq 2 \\ 4ax^3 - 6ax^2 + 9; & x < 2 \end{cases}$, find the value of a that makes $f(x)$ continuous for all real values of x .

(A) -1

(B) 0

(C) 1

(D) 2

(E) 4

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14. $\int x \sin(2x) dx =$

(A) $-\frac{x^2}{2} \cos(2x) + C$

(B) $-\frac{x^2}{4} \cos(2x) + C$

(C) $-\frac{x}{2} \cos(2x) + \frac{1}{4} \sin(2x) + C$

(D) $-\frac{x}{2} \cos(2x) + \frac{1}{2} \cos(2x) + C$

(E) $-\frac{1}{2} \cos(2x) + \frac{1}{4} \sin(2x) + C$

15. If $f(x) = \frac{x^2 + 5x - 24}{x^2 + 10x + 16}$, then $\lim_{x \rightarrow -8} f(x)$ is

(A) 0

(B) 1

(C) $-\frac{3}{2}$

(D) $\frac{11}{6}$

(E) Nonexistent

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16. What is the approximation of the value of e^3 obtained by using the fourth-degree Taylor Polynomial about $x = 0$ for e^x ?

(A) $1 + 3 + \frac{3^2}{2} + 3^2 + \frac{3^4}{4}$

(B) $1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}$

(C) $1 - 3 + \frac{3^2}{2!} - \frac{3^3}{3!} + \frac{3^4}{4!}$

(D) $1 - 3 + \frac{3^2}{2} - 3^2 + \frac{3^4}{4}$

(E) $1 - \frac{3^2}{2!} + \frac{3^4}{4!}$

-
17. A rock is thrown straight upward with an initial velocity of 50 m/s from a point 100 m above the ground.

If the acceleration of the rock at any time t is $a = -10 \text{ m/s}^2$, what is the maximum height of the rock (in meters)?

(A) 125

(B) 150

(C) 175

(D) 200

(E) 225

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18. The sum of the infinite geometric series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \dots$ is

(A) -6

(B) -3

(C) 0

(D) $\frac{3}{7}$

(E) $\frac{3}{2}$

19. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2(5^n)}$ converges?

(A) $-2 \leq x \leq 8$

(B) $-2 < x \leq 8$

(C) $-2 \leq x < 8$

(D) $-5 \leq x \leq 5$

(E) $-5 \leq x < 5$

20. Find the area inside one loop of the curve $r = \sin 2\theta$.


(A) $\frac{\pi}{16}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

(E) π

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21. The average value of $\sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ is

- (A) $\frac{8}{\pi}$
(B) $\frac{12\sqrt{3}-12}{\pi}$
(C) $\frac{12-4\sqrt{3}}{\pi}$
(D) $\frac{6\sqrt{2}-6}{\pi}$
(E) $\frac{6-6\sqrt{2}}{\pi}$
-

22. Find the length of the arc of the curve defined by $x = \frac{1}{2}t^2$ and $y = \frac{1}{9}(6t+9)^{\frac{3}{2}}$, from $t = 0$ to $t = 2$.

- (A) 8 (B) 10 (C) 12 (D) 14 (E) 16
-

GO ON TO THE NEXT PAGE 

23. The function f is given by $f(x) = x^4 + 4x^3$. On which of the following intervals is f decreasing?

- (A) $(-3, 0)$ (B) $(0, \infty)$ (C) $(-3, \infty)$ (D) $(-\infty, -3)$ (E) $(-\infty, 0)$
-

24. Which of the following series converge(s)?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

III. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$

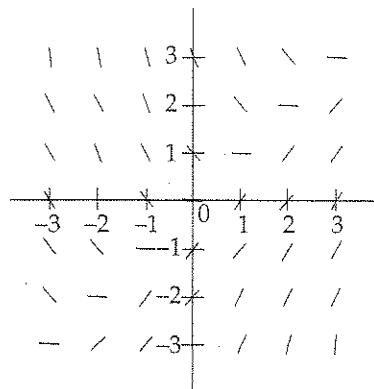
- (A) I only (B) II only (C) I and II (D) I and III (E) I, II, and III
-

25. Given the differential equation $\frac{dz}{dt} = z\left(4 - \frac{z}{100}\right)$, where $z(0) = 50$, what is $\lim_{t \rightarrow \infty} z(t)$?

- (A) 400 (B) 200 (C) 100 (D) 50 (E) 4
-

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26.



The slope field shown above corresponds to which of the following differential equations?

- (A) $\frac{dy}{dx} = \frac{x}{y}$
- (B) $\frac{dy}{dx} = \frac{y}{x}$
- (C) $\frac{dy}{dx} = xy$
- (D) $\frac{dy}{dx} = x - y$
- (E) $\frac{dy}{dx} = x + y$

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27. The value of c that satisfies the Mean Value Theorem for Derivatives on the interval $[0, 5]$ for the function

$$f(x) = x^3 - 6x \text{ is}$$

(A) $-\frac{5}{\sqrt{3}}$

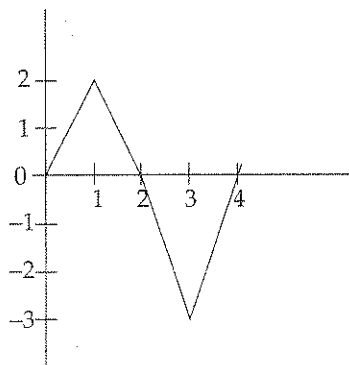
(B) 0

(C) 1

(D) $\frac{5}{3}$

(E) $\frac{5}{\sqrt{3}}$

28.



The graph of f is shown in the figure above. If $g(x) = \int_0^x f(t) dt$, for what positive value of x does $g(x)$ have a minimum?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

STOP

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

CALCULUS BC

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. If $f(x)$ is the function given by $f(x) = e^{3x} + 1$, at what value of x is the slope of the tangent line to $f(x)$ equal to 2?

- (A) $-.135$ (B) 0 (C) $.231$ (D) $-.366$ (E) $.693$

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30. If $y = (\sin x)^{e^x}$, then, when defined, $y' =$

- (A) $(\sin x)^{e^x} (\cos x)$
- (B) $(\cos x)^{e^x}$
- (C) $e^x (\cot x + \ln(\sin x))$
- (D) $e^x (\sin x)^{e^x} (\cot x + \ln(\sin x))$
- (E) $e^x (\sin x)^{e^x} (\cot x)$

31. The side of a square is increasing at a constant rate of 0.4 cm/sec . In terms of the perimeter, P , what is the rate of change of the area of the square, in cm^2/sec ?

- (A) $0.05P$ (B) $0.2P$ (C) $0.4P$ (D) $6.4P$ (E) $51.2P$
-

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32. If f is a vector-valued function defined by $f(t) = (\sin 2t, \sin^2 t)$, then $f''(t) =$

(A) $(-4 \sin 2t, 2 \cos 2t)$

(B) $(-\sin 2t, -\cos^2 t)$

(C) $(4 \sin 2t, \cos^2 t)$

(D) $(4 \sin 2t, -2 \cos 2t)$

(E) $(2 \cos 2t, -4 \sin 2t)$

33. The height of a mass hanging from a spring at time t seconds, where $t > 0$, is given by $h(t) = 12 - 4 \cos(2t)$. In the first two seconds, how many times is the velocity of the mass equal to 0?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

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34. $\lim_{h \rightarrow 0} \frac{\tan^{-1}(1+h) - \frac{\pi}{4}}{h} =$

(A) 2

(B) $\frac{4}{4 + \pi^2}$

(C) $\frac{16}{16 + \pi^2}$

(D) $\frac{1}{2}$

(E) Does Not Exist

35. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407

(B) 13.565

(C) 19.972

(D) 27.879

(E) 34.944

36. Given $x^2y + x^2 = y^2 + 1$, find $\frac{d^2y}{dx^2}$ at $(1,1)$.


(A) 36

(B) 12

(C) -4

(D) -12

(E) -36

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37. If $\int_{-2}^4 f(x) dx = a$ and $\int_3^4 f(x) dx = b$ then $\int_3^{-2} f(x) dx =$

(A) $a + b$

(B) $a - 2b$

(C) $a - b$

(D) $b - a$

(E) $2b - a$

38. $\frac{d}{dx} \int_{2x}^{5x} \cos t \, dt =$

(A) $5 \cos 5x - 2 \cos 2x$

(B) $5 \sin 5x - 2 \sin 2x$

(C) $\cos 5x - \cos 2x$

(D) $\sin 5x - \sin 2x$

(E) $\frac{1}{5} \cos 5x - \frac{1}{2} \sin 2x$

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39. Using the Taylor Series about $x = 0$ for $\sin x$, approximate $\sin(0.4)$ to four decimal places.

(A) $0.4 + \frac{(0.4)^3}{3!} + \frac{(0.4)^5}{5!}$

(B) $0.4 - \frac{(0.4)^3}{3!} + \frac{(0.4)^5}{5!}$

(C) $0.4 - \frac{(0.4)^3}{3} + \frac{(0.4)^5}{5}$

(D) $0.4 + \frac{(0.4)^2}{2!} + \frac{(0.4)^3}{3!} + \frac{(0.4)^4}{4!} + \frac{(0.4)^5}{5!}$

(E) $0.4 - \frac{(0.4)^2}{2!} + \frac{(0.4)^3}{3!} - \frac{(0.4)^4}{4!} + \frac{(0.4)^5}{5!}$

40. Let R be the region in the first quadrant between the graphs of $y = e^{-x}$, $y = \sin x$, and the y -axis. The volume of the solid that results when R is revolved about the x -axis is

(A) -0.888

(B) -0.869

(C) 0.277

(D) 0.869

(E) 0.888

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41. Use Euler's method, with $h = 0.2$ to estimate $y(3)$, if $\frac{dy}{dx} = 2y - 4x$ and $y(2) = 6$.


- (A) 9.684 (B) 10.442 (C) 12.379 (D) 12.756 (E) 18.426
-

42. $\int \sec^4 x \, dx =$

- (A) $\tan^4 x + C$
(B) $\tan x + \frac{1}{3}\tan^3 x + C$
(C) $\tan^2 x + C$
(D) $\frac{\sec^5 x}{5} + C$
(E) $\sec^2 x \tan^2 x + C$
-

43. Let $f(x) = \int \cot x \, dx$; $0 < x < \pi$. If $f\left(\frac{\pi}{6}\right) = 1$, then $f(1) =$

- (A) -1.861 (B) -0.480 (C) 0.134 (D) 0.524 (E) 1.521
-

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44. $\int \sqrt{4-x^2} dx =$

(A) $\frac{2}{3}(4-x^2)^{\frac{3}{2}} + C$

(B) $2\sin^{-1}\left(\frac{x}{2}\right) + x\sqrt{4-x^2} + C$

(C) $2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x}{2}\sqrt{4-x^2} + C$

(D) $\frac{2}{3}(4-x^2)^{\frac{3}{2}} + C$

(E) $2\sin^{-1}\left(\frac{x}{2}\right) + 4x\sqrt{4-x^2} + C$

45. A force of 250 N is required to stretch a spring 5 m from rest. Using Hooke's law, $F = kx$, how much work, in Joules, is required to stretch the spring 7 m from rest?

(A) 14.286

(B) 71.429

(C) 245

(D) 490

(E) 1225

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Percent of total grade—50

Part A: 45 minutes—3 problems

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Part B: 45 minutes, 3 problems

(No calculator is allowed for these problems.)

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator

GENERAL INSTRUCTIONS FOR SECTION II PART A AND SECTION II PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED

- Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly label functions, graphs, tables, or other objects you use.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If a calculation is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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CALCULUS BC

SECTION II, Part A

Time—45 Minutes

Number of questions—3

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS

1. An object moving along a curve in the xy -plane has its position given by $(x(t), y(t))$ at time t seconds,

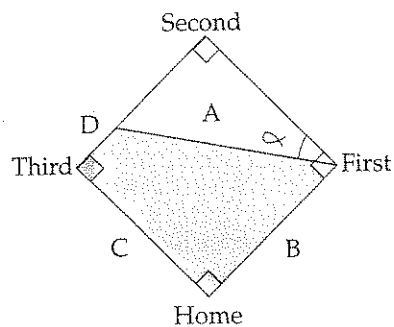
$$0 \leq t \leq 1, \text{ with } \frac{dx}{dt} = 8t \cos t \text{ units/sec and } \frac{dy}{dt} = 8t \sin t \text{ units/sec.}$$

At time $t = 0$, the object is located at $(5, 11)$

- (a) Find the speed of the object at time $t = 1$.
- (b) Find the length of the arc described by the curve's position from time $t = 0$ to time $t = 1$.
- (c) Find the location of the object at time $t = \frac{\pi}{2}$.
-

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2.



A baseball diamond is a square with each side 90 feet in length. A player runs from second base to third base at a rate of 18 ft/sec.

- (a) At what rate is the player's distance from first base, A , changing when his distance from third base, D , is 22.5 feet?
- (b) At what rate is angle α increasing when D is 22.5 feet?
- (c) At what rate is the area of the trapezoidal region, formed by line segments A , B , C , and D , changing when D is 22.5 feet?

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3. A body is coasting to a stop and the only force acting on it is a resistance proportional to its speed, according to the equation $\frac{ds}{dt} = v_f = v_0 e^{-\left(\frac{k}{m}\right)t}$; $s(0) = 0$, where v_0 is the body's initial velocity (in m/s), v_f is its final velocity, m is its mass, k is a constant, and t is time.

(a) If a body with mass $m = 50\text{kg}$ and $k = 1.5\text{kg/sec}$, initially has a velocity of 30 m/s , how long, to the nearest second, will it take to slow to 1 m/s ?

(b) How far, to the nearest 10 meters, will the body coast during the time it takes to slow from 30 m/s to 1 m/s ?

(c) If the body coasts from 30 m/s to a stop, how far will it coast?

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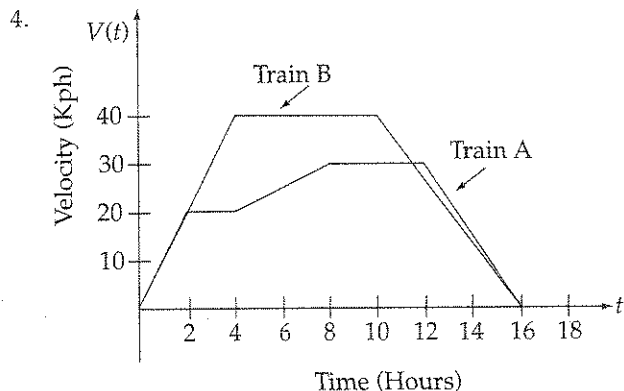
CALCULUS BC

SECTION II, Part B

Time—45 Minutes

Number of questions—3

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS



Three trains, A , B , and C each travel on a straight track for $0 \leq t \leq 16$ hours. The graphs above, which consist of line segments, show the velocities, in kilometers per hour, of trains A and B . The velocity of C is given by $v(t) = 8t - 0.25t^2$

(Indicate units of measure for all answers.)

- Find the velocities of A and C at time $t = 6$ hours.
- Find the accelerations of B and C at time $t = 6$ hours.
- Find the positive difference between the total distance that A traveled and the total distance that B traveled in 16 hours.
- Find the total distance that C traveled in 16 hours.

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5. Let y be the function satisfying $f'(x) = x(1 - f(x))$; $f(0) = 10$.

(a) Use Euler's method, starting at $x = 0$, with step size of 0.5 to approximate $f(x)$ at $x = 1$.

(b) Find an exact solution for $f(x)$, in terms of x , when $x = 1$.

(c) Evaluate $\int_0^{\infty} x(1 - f(x))dx$

6. Given $f(x) = \tan^{-1}(x)$ and $g(x) = \frac{1}{1+x}$, for $|x| \leq 1$.

(a) Find the fifth-degree Taylor Polynomial and general expression for $g(x)$ about $x = 0$.

(b) Given that $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$, for $|x| \leq 1$, use the result of part (a) to find the fifth-degree Taylor Polynomial and general expression for $f(x)$ about $x = 0$.

(c) Use the fifth-degree Taylor Polynomial to estimate $f\left(\frac{1}{10}\right)$.

END OF EXAMINATION